## Lesson 21

Objective: Explain the size of the product, and relate fraction and decimal equivalence to multiplying a fraction by 1.

## Suggested Lesson Structure

| $\square$ | Fluency Practice |
| :--- | :--- |
| $\square$ | (12 minutes) |
| Application Problem | (7 minutes) |
| Concept Development | (31 minutes) |
| $\square$ Student Debrief | (10 minutes) |
| Total Time | (60 minutes) |



## Fluency Practice (12 minutes)

- Sprint: Multiply Decimals 5.NBT. 7 (8 minutes)
- Find the Unit Conversion 5.MD. 2


## Sprint: Multiply Decimals (8 minutes)

Materials: (S) Multiply Decimals Sprint
Note: This fluency activity reviews Lessons 17-18.

## Find the Unit Conversion (4 minutes)

Materials: (S) Personal white board
Note: This fluency activity reviews Lesson 20.
T: (Write $2 \frac{1}{3} \mathrm{yd}=$ $\qquad$ ft.) How many feet are in 1 yard?

S: 3 feet.
T: Express $2 \frac{1}{3}$ yards as an improper fraction.
S: $\quad \frac{7}{3}$ yards.
T: Write an expression using the improper fraction and feet. Then, solve.
S: (Write $\frac{7}{3} \times 3$ feet $=7$ feet.)
$2 \frac{1}{3} y d=$ _I $H$
$=2 \frac{1}{3} \times 1 \mathrm{yd}$
$=\frac{7}{3} \times 3 \mathrm{ff}$
$=\frac{21}{3} \mathrm{ft}$
$=7 \mathrm{ft}$

T: $\quad 2 \frac{1}{3}$ yards equals how many feet? Answer in a complete sentence.
S: $\quad 2 \frac{1}{3}$ yards equals 7 feet.

Continue with one or more of the following possible problems: $2 \frac{1}{4} \mathrm{gal}=$ $\qquad$ qt, $2 \frac{3}{4} \mathrm{ft}=$ $\qquad$ in, and $7 \frac{1}{2} \mathrm{pt}=$ $\qquad$ c.

## Application Problem (7 minutes)

Carol had $\frac{3}{4}$ yard of ribbon. She wanted to use it to decorate two picture frames. If she uses half the ribbon on each frame, how many feet of ribbon will she use for one frame? Use a tape diagram to show your thinking.

Note: This Application Problem draws on fraction multiplication concepts taught in previous lessons in this module.


She will use $1 \frac{1}{8}$ feet of ribbon for one frame.

## Concept Development (31 minutes)

Materials: (S) Personal white board
Problem 1: $\frac{2}{2}$ of $\frac{3}{4}$
T: (Post Problem 1 on the board.) Write a multiplication expression for this problem.
S: (Write $\frac{2}{2} \times \frac{3}{4}$.)
T: Work with a partner to find the product of 2 halves and 3 fourths.
S: (Work.)


T: Say the product.
S: $\frac{6}{8}$.
$\frac{2}{2}$ of $\frac{3}{4}=\frac{6}{8}$
T: (Write $=\frac{6}{8}$.) Let's draw an area model to verify our solution. (Draw a rectangle and label it 1.) What are we taking 2 halves of?
S: $\frac{3}{4}$.
T: (Partition the model vertically into fourths and shade 3 of them.) How do we show 2 halves?
S: Split each fourth unit into 2 equal parts, and shade both of them.
T: (Partition fourths horizontally, and shade both halves, or 6 eighths.) What is the product?
S: 6 eighths.
T: How does the size of the product, $\frac{6}{8}$, compare to the size of the original fraction, $\frac{3}{4}$ ? Turn and talk.

S: They're exactly same amount. 6 eighths and 3 fourths are equal. $\rightarrow$ They're the same. 3 fourths is just 6 eighths in simplest form. $\rightarrow$ Eighths are a smaller unit than fourths, but we have twice as many of them, so really, the two fractions are equal.
T: I hear you saying that the product, $\frac{6}{8}$, is equal to the amount we had at first, $\frac{3}{4}$. We multiplied. How is it possible that our quantity has not changed? Turn and talk.
S: We multiplied by 2 halves, which is a whole. So, I'm thinking we showed $\frac{3}{4}$ just using a different name. $\rightarrow 2$ halves is equal to 1 , so really, we just multiplied 3 fourths by 1 . Anything times 1 will result in itself. $\rightarrow$ The fraction two-over-two is equivalent to 1 . We just created an equivalent fraction by multiplying the numerator and denominator by a common factor.
T: It sounds like you think that our beginning amount (point to $\frac{3}{4}$ ) didn't change because we multiplied by one. Name some other fractions that are equal to 1 .
S: 3 thirds. $\rightarrow 4$ fourths. $\rightarrow 10$ tenths. $\rightarrow 1$ million millionths!
T: Let's test your hypothesis. Work with a partner to find $\frac{3}{3}$ of $\frac{3}{4}$. One of you can multiply the fractions, while the other draws an area model.
S: (Work and share.)
MP. 3 T: What did you find out?
S: It happened again. The product is 9 twelfths, which is still equal to 3 fourths. $\rightarrow$ We were right: 3 thirds is equal to 1 , so we got another product that is equal to 3 fourths. $\rightarrow$ My area model shows it very clearly. Even though twelfths are a smaller unit, 9 twelfths is equal to 3 fourths.
T: Show some other fraction multiplication expressions involving 3 fourths that would give us a product equal in size to 3 fourths.
S: (Show $\frac{3}{4} \times \frac{5}{5} . \rightarrow \frac{8}{8} \times \frac{3}{4} . \rightarrow \frac{100}{100} \times \frac{3}{4}$.)
T: Is $\frac{3}{4}$ equal to $\frac{18}{24}$ ? Turn and talk.
S: Yes, if we multiplied 3 fourths by 6 sixths, we'd get $\frac{18}{24}$. $\rightarrow$ Sure, $\frac{3}{4}$ is $\frac{18}{24}$ in simplest form. I can divide 18 and 24 by 6.
T: Is $\frac{1}{4}$ equal to $\frac{25}{100}$ ? Work with a partner to write a multiplication sentence and share your thinking.
S: Yes. I know 25 cents is 1 fourth of 100 cents. $\rightarrow$ It is equal because if we multiply 1 fourth and 25 twenty-fifths, that renames the same amount just using hundredths. It's like all the others we've done today.

$$
\frac{1}{4} \times \frac{25}{25}=\frac{25}{100}
$$

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Problem 2: Express fractions as an equivalent decimal.
a. $\frac{1}{5} \times \frac{2}{2}=0.2$
b. $\frac{1}{4}=0.250=0.25$
C. $\frac{1}{8}=0.125$

T (Write Problem 1(a), $\frac{1}{5} \times \frac{2}{2}$, on the board.) Show the product.
S: $\frac{2}{10}$.
T: (Write $=\frac{2}{10}$.) What are some other ways to express $\frac{2}{10}$ ? Turn and talk.


S: We could write it in unit form, like 2 tenths. $\rightarrow$ One-fifth. $\rightarrow$ Tenths. That's a decimal. We could write it as 0.2 .
$\mathrm{T}: \quad$ Express $\frac{2}{10}$ as a decimal on your personal white board.
S: (Write 0.2.)
T: (Write $=0.2$.) We multiplied one-fifth by a fraction equal to 1 . Did that change the value of onefifth?

S: No.
T : So, if $\frac{1}{5}$ is equal to $\frac{2}{10}$, and $\frac{2}{10}$ is equal to 0.2 . Can we say that $\frac{1}{5}=0.2$ ? (Write $\frac{1}{5}=0.2$.) Turn and talk.
S: They are the same. We multiplied one-fifth by 1 to reach $\frac{2}{10}$, so they must be the same.
T: Let's try 3 fifths. How can we change 3 fifths to a decimal?
S: We could multiply by $\frac{2}{2}$ again. $\rightarrow$ Since we know one-fifth is equal to $0.2,3$ fifths is just 3 times more than that, so we could triple 0.2.
T: Work with a partner to express 3 fifths as a decimal.

## NOTES ON <br> MULTIPLE MEANS OF REPRESENTATION:

Once students are comfortable renaming fractions using decimal units, make a connection to the powers of 10 concepts learned back in Module 1. Students can be challenged to see that tenths can be notated as $\frac{1}{10^{1}}$, hundredths as $\frac{1}{10^{2}}$, and thousandths as $\frac{1}{10^{3}}$.

S: (Work and share.)
T: Say $\frac{3}{5}$ as a decimal.
S: 0.6.
T: (Write Problem 2(b), $\frac{1}{4}$, on the board.) All of the fractions we have worked with so far have been related to tenths. Let's think about 1 fourth. We just agreed a moment ago that 1 fourth was equal to 25 hundredths. Write 25 hundredths as a decimal.

S: 0.25.

T: Fourths were renamed as hundredths in this decimal. Could we have easily renamed fourths as tenths? Why or why not? Turn and talk.
S: We can't rename fourths as tenths because 4 isn't a factor of $10 . \rightarrow$ There's no whole number we can use to get from 4 to 10 using multiplication. $\rightarrow$ We could name 1 fourth as tenths, but that would be 2 and a half tenths, which is weird.
T : Since tenths are not possible, what unit did we use, and how did we get there?
S: We used hundredths. $\rightarrow$ We multiplied by 25 twenty-fifths.
T: Is 25 hundredths the only decimal name for 1 fourth? Is there another unit that would rename fourths as a decimal? Turn and talk.
S: We could multiply 25 hundredths by 10 tenths; that would be 250 thousandths. So, we could do it in two steps. $\rightarrow$ If we multiply 1 fourth by 250 over 250 , that would get us to 250 thousandths. $\rightarrow$ Four 250s is equal to a thousand.
T : Work with a neighbor to express $\frac{1}{4}$ as a decimal, showing your work with multiplication sentences. One of you multiply by $\frac{25}{25}$, and the other multiply by $\frac{250}{250}$. Compare your work when you're done.
S: (Work and share.)
T: What did you find? Are the products the same?
S: Some of us got 25 hundredths, and some of us got 250 thousandths. They look different, but they're equal. I got 0.25 , which looks like 25 cents; that's a quarter. Wow, that must be why we call $\frac{1}{4}$ a quarter!
T: (Write $\frac{1}{4}=0.250=0.25$.) What about $\frac{2}{4}$ ? How could we express that as a decimal? Tell a neighbor what you think, and then show $\frac{2}{4}$ as a decimal.
S: We could multiply by $\frac{25}{25}$ again. $\rightarrow 2$ fourths is a half. 1 half is $0.5 . \rightarrow 2$ fourths is twice as much as 1 fourth. We could just double 0.25 . (Show $\frac{2}{4}=0.5$.)
T: (Write Problem 2(c), $\frac{1}{8}$, on the board.) Are eighths a unit we can express directly as a decimal, or do we need to multiply by a fraction equal to 1 first?
S : We'll need to multiply first.


T: What fraction equal to 1 will help us rename eighths? Discuss with your neighbor.
S: Eight isn't a factor of 10 or 100. I'm not sure. $\rightarrow$ I don't know if 1,000 can be divided by 8 without a remainder. I'll divide. Hey, it works!
T : Jonah, what did you find out?
S: $1,000 \div 8=125$. We can multiply by $\frac{125}{125}$.
T: Work independently, and try Jonah's strategy. Show your work when you're done.
S: (Work and show $\frac{1}{8}=0.125$.)
T: How would you express $\frac{2}{8}$ as a decimal? Tell a neighbor.

S: We could multiply by $\frac{125}{125}$ again. $\rightarrow$ We could just double 0.125 and get $0.250 . \rightarrow \frac{2}{8}$ is equal to $\frac{1}{4}$. We already solved that as 0.250 or 0.25 .
T: Work independently to show $\frac{2}{8}$ as a decimal.
S: (Show $\frac{2}{8}=0.250$ or $\frac{2}{8}=0.25$.)
T : It's a good idea to remember some of these common fraction-decimal equivalencies, such as fourths and eighths; you will use them often in your future math work.
Continue with the following possible sequence: $\frac{1}{20}, 1 \frac{1}{20}$, $\frac{6}{25}$, and $\frac{51}{50}$.

## NOTES ON <br> PROPERTIES <br> OF OPERATIONS:

After completing this lesson, it may be interesting to some students to know the name of the property they have been studying: multiplicative identity property of 1. Consider asking students if they can think of any other identity properties. Ideally, they will say that zero added to any number keeps the same value. This is the additive identity property of 0 . (See Table 3 of the Common Core Learning Standards.)

## Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

## Student Debrief (10 minutes)

Lesson Objective: Explain the size of the product, and relate fraction and decimal equivalence to multiplying a fraction by 1.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for
 misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- Share your response to Problem 1(d) with a partner.
- In Problem 2, what is the relationship between Parts (a) and (b), (c) and (d), (e) and (f), (i) and (k), and (j) and (I)? (They have the same denominator.)

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- In Problem 2, what did you notice about Parts (f), (g), (h), and (j)? (The fractions are greater than 1; thus, the answers will be more than one whole.)
- In Problem 2, what did you notice about Parts (k) and (I)? (The fractions are mixed numbers; thus, the answers will be more than one whole.)
- Share and explain your thought process for answering Problem 3.
- In Problem 4, did you have the same expressions to represent one on the number line as your partner's? Can you think of more expressions?
- How did you solve Problem 5? Share your strategy and solution with a partner.


## Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

3. Jack said that if you take a number and multiply it by a fraction, the product will always be smaller than What you started with. Is he correct? Why or why not? Explain your answer and give at least two
examples to support your thinking.
$\begin{array}{ll}\text { Jack is right some of } \\ \text { the time, but not always. } & 3 \times \frac{2}{2}=\frac{6}{2}=3 \quad 3 \times \frac{4}{4}=\frac{12}{4}=3\end{array}$
If you multiply by a
fraction equal to 1 (like $\frac{2}{2}$ )
the product isequal to what you started with.
4. There is an infinite number of ways to represent 1 on the number line. In the space below, write at least four expressions multiplying by 1 . Represent "one" differently in each expression.

$$
8 \times \frac{1}{1}=\frac{8}{1}=8 \quad 5 \times \frac{10}{10}=\frac{50}{10}=5
$$

$$
12 \times \frac{2}{2}=\frac{24}{2}=12 \quad 100 \times \frac{100}{100}=\frac{10,000}{100}=100
$$

5. Maria multiplied by one to rename $\frac{1}{4}$ as hundredths. She made factor pairs equal to 10 . Use her method
to change one-eighth to an equivalent decimal.
Maria's way: $\frac{1}{4}=\frac{1}{2 \times 2} \times \frac{5 \times 5}{5 \times 5}=\frac{5 \times 5}{(2 \times 5) \times(2 \times 5)}=\frac{25}{100}=0.25$
$\frac{1}{8}=\frac{1}{2 \times 2 \times 2}=\frac{5 \times 5 \times 5}{5 \times 5 \times 5}=\frac{5 \times 5 \times 5}{(2 \times 5)(2 \times 5)(2 \times 5)}=\frac{125}{1,000}=0.125$

as $\frac{1}{4}$. Can you use his ideas to show another:wz; :0 find the dectma! equal to $\frac{1}{8}$ ?
$\frac{1}{4}=0.25=0.250=250$ thousandths
$\frac{1}{8}$ is half of $\frac{1}{4}$. Half of 250 thousandths is 125 thousandths.
$\frac{1}{8}=0.125$


| A |
| :--- |
| Multiply      <br> 1 $3 \times 2=$  23 $0.6 \times 2=$  <br> 2 $3 \times 0.2=$  24 $0.6 \times 0.2=$  <br> 3 $3 \times 0.02=$  25 $0.6 \times 0.02=$  <br> 4 $3 \times 3=$  26 $0.2 \times 0.06=$  <br> 5 $3 \times 0.3=$  27 $5 \times 7=$  <br> 6 $3 \times 0.03=$  28 $0.5 \times 7=$  <br> 7 $2 \times 4=$  29 $0.5 \times 0.7=$  <br> 8 $2 \times 0.4=$  30 $0.5 \times 0.07=$  <br> 9 $2 \times 0.04=$  31 $0.7 \times 0.05=$  <br> 10 $5 \times 3=$  32 $2 \times 8=$  <br> 11 $5 \times 0.3=$  33 $9 \times 0.2=$  <br> 12 $5 \times 0.03=$  34 $3 \times 7=$  <br> 13 $7 \times 2=$  35 $8 \times 0.03=$  <br> 14 $7 \times 0.2=$  36 $4 \times 6=$  <br> 15 $7 \times 0.02=$  37 $0.6 \times 7=$  <br> 16 $4 \times 3=$  38 $0.7 \times 0.7=$  <br> 17 $4 \times 0.3=$  39 $0.8 \times 0.06=$  <br> 18 $0.4 \times 3=$  40 $0.09 \times 0.6=$  <br> 19 $0.4 \times 0.3=$  41 $6 \times 0.8=$  <br> 20 $0.4 \times 0.03=$  42 $0.7 \times 0.9=$  <br> 21 $0.3 \times 0.04=$  43 $0.08 \times 0.8=$  <br> 22 $6 \times 2=$  44 $0.9 \times 0.08=$  |

B
Improvement $\qquad$ \# Correct $\qquad$
Multiply.

| 1 | $4 \times 2=$ |  | 23 | $0.8 \times 2=$ |  |
| :---: | :---: | :--- | :---: | :---: | :---: |
| 2 | $4 \times 0.2=$ |  | 24 | $0.8 \times 0.2=$ |  |
| 3 | $4 \times 0.02=$ |  | 25 | $0.8 \times 0.02=$ |  |
| 4 | $2 \times 3=$ |  | 26 | $0.2 \times 0.08=$ |  |
| 5 | $2 \times 0.3=$ |  | 27 | $5 \times 9=$ |  |
| 6 | $2 \times 0.03=$ |  | 28 | $0.5 \times 9=$ |  |
| 7 | $3 \times 3=$ |  | 29 | $0.5 \times 0.9=$ |  |
| 8 | $3 \times 0.3=$ |  | 30 | $0.5 \times 0.09=$ |  |
| 9 | $3 \times 0.03=$ |  | 31 | $0.9 \times 0.05=$ |  |
| 10 | $4 \times 3=$ |  | 32 | $2 \times 6=$ |  |
| 11 | $4 \times 0.3=$ |  | 33 | $7 \times 0.2=$ |  |
| 12 | $4 \times 0.03=$ |  | 34 | $3 \times 8=$ |  |
| 13 | $9 \times 2=$ |  | 35 | $9 \times 0.03=$ |  |
| 14 | $9 \times 0.2=$ |  | 36 | $4 \times 8=$ |  |
| 15 | $9 \times 0.02=$ |  | 37 | $0.7 \times 6=$ |  |
| 16 | $5 \times 3=$ |  | 38 | $0.6 \times 0.6=$ |  |
| 17 | $5 \times 0.3=$ |  | 39 | $0.6 \times 0.08=$ |  |
| 18 | $0.5 \times 3=$ |  | 40 | $0.06 \times 0.9=$ |  |
| 19 | $0.5 \times 0.3=$ |  | 41 | $8 \times 0.6=$ |  |
| 20 | $0.5 \times 0.03=$ |  | 42 | $0.9 \times 0.7=$ |  |
| 21 | $0.3 \times 0.05=$ |  | 43 | $0.07 \times 0.7=$ |  |
| 22 | $8 \times 2=$ |  | 44 | $0.8 \times 0.09=$ |  |

Name $\qquad$ Date $\qquad$

1. Fill in the blanks. The first one has been done for you.
a. $\frac{1}{4} \times 1=\frac{1}{4} \times \frac{3}{3}=\frac{3}{12}$
b. $\frac{3}{4} \times 1=\frac{3}{4} \times-=\frac{21}{28}$
c. $\frac{7}{4} \times 1=\frac{7}{4} \times-=\frac{35}{20}$
d. Use words to compare the size of the product to the size of the first factor.
2. Express each fraction as an equivalent decimal.
a. $\frac{1}{4} \times \frac{25}{25}=$
b. $\frac{3}{4} \times \frac{25}{25}=$
c. $\frac{1}{5} \times-=$
d. $\frac{4}{5} \times-=$
e. $\frac{1}{20}$
f. $\frac{27}{20}$
g. $\frac{7}{4}$
h. $\frac{8}{5}$
i. $\frac{24}{25}$
j. $\frac{93}{50}$
k. $2 \frac{6}{25}$
I. $3 \frac{31}{50}$

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3. Jack said that if you take a number and multiply it by a fraction, the product will always be smaller than what you started with. Is he correct? Why or why not? Explain your answer, and give at least two examples to support your thinking.
4. There is an infinite number of ways to represent 1 on the number line. In the space below, write at least four expressions multiplying by 1 . Represent one differently in each expression.
5. Maria multiplied by 1 to rename $\frac{1}{4}$ as hundredths. She made factor pairs equal to 10 . Use her method to change one-eighth to an equivalent decimal.

$$
\begin{aligned}
& \text { Maria's way: } \frac{1}{4}=\frac{1}{2 \times 2} \times \frac{5 \times 5}{5 \times 5}=\frac{5 \times 5}{(2 \times 5) \times(2 \times 5)}=\frac{25}{100}=0.25 \\
& \frac{1}{8}=
\end{aligned}
$$

Paulo renamed $\frac{1}{8}$ as a decimal, too. He knows the decimal equal to $\frac{1}{4}$, and he knows that $\frac{1}{8}$ is half as much as $\frac{1}{4}$. Can you use his ideas to show another way to find the decimal equal to $\frac{1}{8}$ ?

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1. Fill in the blanks to make the equation true.

$$
\frac{9}{4} \times 1=\frac{9}{4} \times-=\frac{45}{20}
$$

2. Express the fractions as equivalent decimals.
a. $\frac{1}{4}=$
b. $\frac{2}{5}=$
c. $\frac{3}{25}=$
d. $\frac{5}{20}=$

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1. Fill in the blanks.
a. $\frac{1}{3} \times 1=\frac{1}{3} \times \frac{3}{3}=\frac{-}{9}$
b. $\frac{2}{3} \times 1=\frac{2}{3} \times-=\frac{14}{21}$
c. $\frac{5}{2} \times 1=\frac{5}{2} \times-=\underline{25}$
d. Compare the first factor to the value of the product.
2. Express each fraction as an equivalent decimal. The first one is partially done for you.
a. $\frac{3}{4} \times \frac{25}{25}=\frac{3 \times 25}{4 \times 25}=\frac{}{100}=$
b. $\frac{1}{4} \times \frac{25}{25}=$
c. $\frac{2}{5} \times-=$
d. $\frac{3}{5} \times-=$
e. $\frac{3}{20}$
f. $\frac{25}{20}$
g. $\frac{23}{25}$
h. $\frac{89}{50}$
i. $3 \frac{11}{25}$
j. $5 \frac{41}{50}$
3. $\frac{6}{8}$ is equivalent to $\frac{3}{4}$. How can you use this to help you write $\frac{6}{8}$ as a decimal? Show your thinking to solve.
4. A number multiplied by a fraction is not always smaller than the original number. Explain this and give at least two examples to support your thinking.
5. Elise has $\frac{3}{4}$ of a dollar. She buys a stamp that costs 44 cents. Change both numbers into decimals, and tell how much money Elise has after paying for the stamp.
