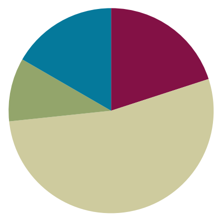
Lesson 3

Objective: Compose and decompose right rectangular prisms using layers.

Suggested Lesson Structure

Fluency Practice	(12 minutes)
Application Problem	(6 minutes)
Concept Development	(32 minutes)
Student Debrief	(10 minutes)
Total Time	**(60 minutes)**



Fluency Practice (12 minutes)

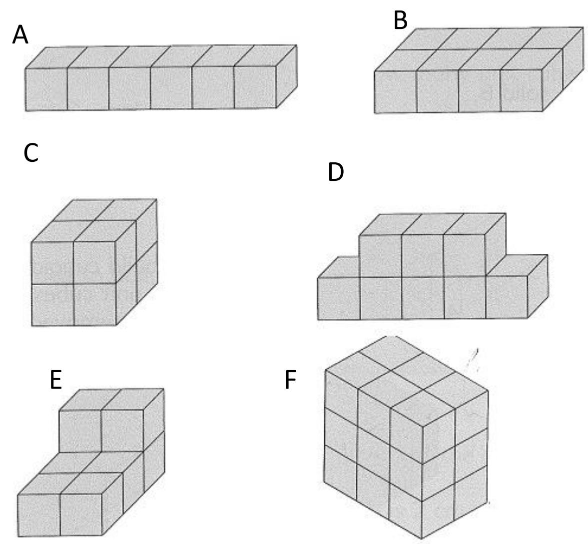
- Sprint: Multiply a Fraction and Whole Number **5.NF.3** (8 minutes)
- Find the Volume **5.MD.3** (4 minutes)

Sprint: Multiply a Fraction and Whole Number (8 minutes)

Materials: (S) Multiply a Fraction and Whole Number Sprint

Note: This Sprint reviews content from G5–M4–Lessons 6–8.

Find the Volume (4 minutes)



Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 1–2.

T: (Project Image A, pictured at right.) Each cube is 1 cubic centimeter. How many cubes are there? Respond on your personal white board.

S: 6.

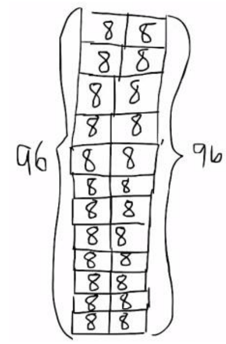
T: Write the volume on your personal white board with the correct units.

S: 6 cubic centimeters.

Follow this sequence for the other images pictured to the right.

Application Problem (6 minutes)

An ice cube tray has two rows of 8 cubes in each. How many ice cubes are in a stack of 12 ice cube trays? Draw a picture to explain your reasoning.



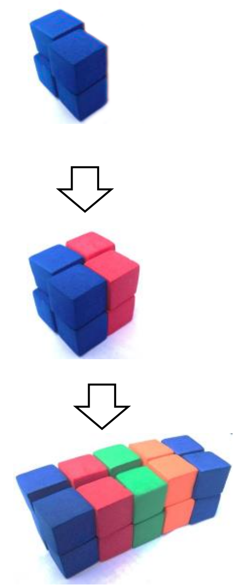
$2 \times 96 = 192$

There are 192 ice cubes.

Note: This Application Problem encourages students to visualize layers in the stack that will be helpful as students refine their understanding of volume in today's lesson.

Concept Development (32 minutes)

Materials: (T) 27 centimeter cubes (S) 27 centimeter cubes, rectangular prism recording sheet (Template)



T: Build this with your own cubes. (Show 4 cubes in a square formation stacked vertically—2 layers with 2 cubes in each layer.)

T: What's the volume of this rectangular prism?

S: 4 cubic centimeters.

T: Let's add layers horizontally. Add another layer *next* to the first one.

S: (Work.)

T: What is the volume?

S: 8 cubic centimeters.

T: Add 3 more layers next to the first two. (Pause for students to do this.)

T: What is the volume now?

S: 20 cubic centimeters.

T: How did you figure that out? Turn and talk.

S: I added the first 8 to the 12 more that I added. → I saw 5 along the bottom, and there were 2 layers going back, so that makes 10, and 2 layers going up makes 20. → I knew that I had 27 cubes to start, and I only have 7 left.

T: (Project a blank rectangular prism from the recording sheet, or draw one on the board.) Let's record how we built the layers. Use the first rectangle in the row of your recording sheet.

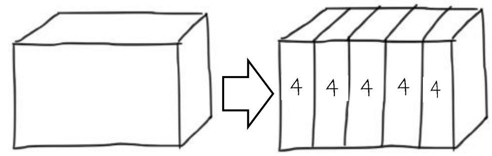
T: How many layers did we build in all?

S: 5.

T: Let's show that by partitioning the prism into 5 layers. Partition the prism vertically into 5 equal sections.) Make your prism look like mine. How many cubes were in each layer?

S: 4 cubes.

T: Record that on each layer that we drew. (Write a 4 on each of the vertical layers.) Write a number sentence that expresses the volume of this prism using these layers. Turn and talk.



$4 cm^3 + 4 cm^3 + 4 cm^3 + 4 cm^3 + 4 cm^3 = 20 cm^3$

5×4 Cubic cm = 20 Cubic cm

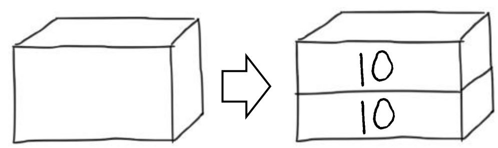
S: We could write 4 cm^3 + 4 cm^3 + 4 cm^3 + 4 cm^3 + 4 cm^3 = 20 cm^3. → Since all the layers are the same, we could write 5 × 4 cubic cm = 20 cubic cm.

T: (Draw the table on the board.) I'll record that in a table. Now, imagine that we could partition this prism into layers like a cake, like our ice cube trays. What might that look like? Work with your partner to show the layers on the next prism in the row, and tell how many cubes would be in each. Use your cubes to help you.

Number of Layers	Cubes in Each Layer	Volume
5	4	20 cm^3
2	10	20 cm^3
2	10	20 cm^3

MP.4

S: The prism is 2 units high, so we could cut the prism in half horizontally from left to right. That would be 10 cubes in each one. → We could make a top layer of 10 cubes and a bottom layer of 10 cubes.



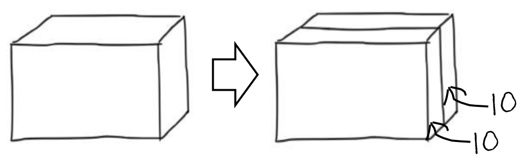
$10 cm^3 + 10 cm^3 = 20 cm^3$

2×10 cubic cm = 20 cubic cm

T: Let's record your thinking. (Draw the figure to the right.) Write a number sentence that expresses the volume of the prism using these layers.

S: 10 cm^3 + 10 cm^3 = 20 cm^3.
→ 2 × 10 cubic cm = 20 cubic cm.

T: Let's record that information in our table. (Record.) Work with your partner to find one last way that we can partition this prism into layers. Use the third prism on your recording sheet to label the layers, and write the number of cubes in each layer. Then, write a number sentence to explain your thinking.



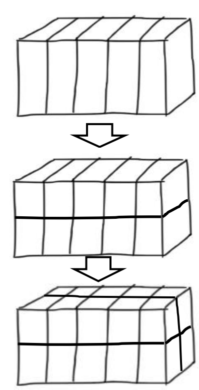
$10 cm^3 + 10 cm^3 = 20 cm^3$

2×10 cubic cm = 20 cubic cm.

S: (Work to draw the third figure and write the number sentences.)

T: I'll record this last bit of information in our table. (Record.)

T: Now, let's draw the different layers together. Use the last prism in the row of your recording sheet.



Step 1: Draw vertical lines to show the 5 layers of 4 cubes each that remind us of bread slices. (Point to table's first line.)

Step 2: Draw a horizontal line to show the two layers of 10 cubes each that remind us of layers of cake. (Point to table's second line.)

Step 3: Draw both a horizontal and a vertical line to show the front and back layers of 10 each. (Point to table's last line.)

T: What is the volume of the prism?

S: 20 cubic centimeters.

T: Build a prism with a partner that has one 3 cube by 3 cube layer. (Demonstrate building this with cubes.)

T: What is the volume?

S: 9 cubic centimeters.

T: Add another layer of cubes on top.

T: What is the volume now? How do you know?

S: It's 18 cubic centimeters because now, we have 2 groups of 9 cubic centimeters. → Two layers with 9 cubes each is 18 cubic centimeters.

T: Now, add another layer. What is the volume?

S: 27 cubic centimeters.

T: What is the overall shape of your rectangular prism?

S: A cube!

T: Use the set of cubes on your recording sheet to show the three ways of layering using the same system we just did with our 2 by 2 by 5 rectangular prism.

S: (Work.)

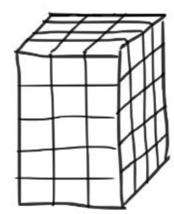
T: (Project or draw an image of a 3 × 4 × 5 rectangular prism. Direct students to the set of vertical prisms on the rectangular prism recording sheet.) Imagine what the bottom layer of this prism would look like. Describe it to your partner, and then build it.

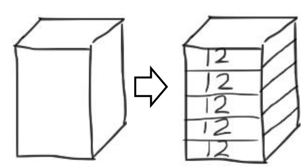
S: There would be 3 rows with 4 cubes in each row. → There would be 12 cubes in all. It would be 3 cubes wide and 4 cubes long and 1 cube high. → This would be like a 4 by 3 rectangle, but it is 1 centimeter tall. (Build.)

T: Here's the same prism but without the unit cubes drawn. How might we represent the bottom layer on this picture? Use your recording sheet, and talk to your partner.

NOTES ON MULTIPLE MEANS OF ENGAGEMENT:

Challenge students who quickly grasp the decompositions by asking them to determine a "rule" for finding the volume and test it for different rectangular prisms. They might also be asked to calculate the volume of the prisms as if they were built from 2 cm cubes. Ask them to explain what would happen to the volume if the dimensions of the cubes were doubled or tripled.





$12\,cm^3 + 12\,cm^3 + 12\,cm^3 + 12\,cm^3 + 12\,cm^3 = 60\,cm^3$

5×12 cubic cm $= 60$ cubic cm

S: We could draw a horizontal slice toward the bottom and label it with 12. → I can see in the drawing that there are 5 layers in all, so I'll need to make the bottom about 1 fifth of the prism and put 12 on it.

T: What is the volume of the single layer?

S: 12 cubic centimeters.

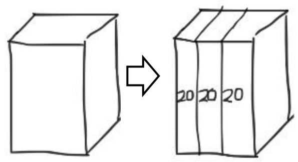
T: What is the volume of the prism with 5 of these layers?

S: I know there are 5 layers that are the same, so 12 cm^3 + 12 cm^3 + 12 cm^3 + 12 cm^3 + 12 cm^3, so 60 cm^3. → It's 5 × 12 cubic cm, so 60 cubic cm.

T: What other ways could we partition this prism into layers? Turn and talk, and then draw a picture of your thinking on the recording sheet.

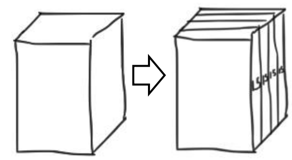
S: (Draw.)

Possible Solutions



$20cm^3 + 20cm^3 + 20cm^3 = 60cm^3$

3×20 cubic cm = 60 cubic cm

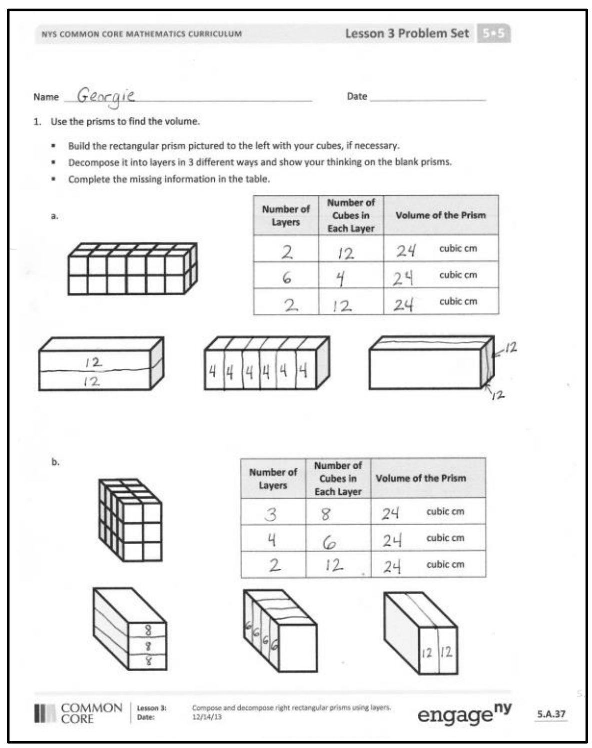


$15cm^3 + 15cm^3 + 15cm^3 + 15cm^3 = 60cm^3$

4×15 cubic cm = 60 cubic cm

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Problem Set 5•5

Name Georgie Date

1. Use the prisms to find the volume.
   - Build the rectangular prism pictured to the left with your cubes, if necessary.
   - Decompose it into layers in 3 different ways and show your thinking on the blank prisms.
   - Complete the missing information in the table.

a.

Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
2	12	24 cubic cm
6	4	24 cubic cm
2	12	24 cubic cm

b.

Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
3	8	24 cubic cm
4	6	24 cubic cm
2	12	24 cubic cm

COMMON CORE Lesson 3: Compose and decompose right rectangular prisms using layers. Date: 12/14/13 engage ny 5.A.37

Student Debrief (10 minutes)

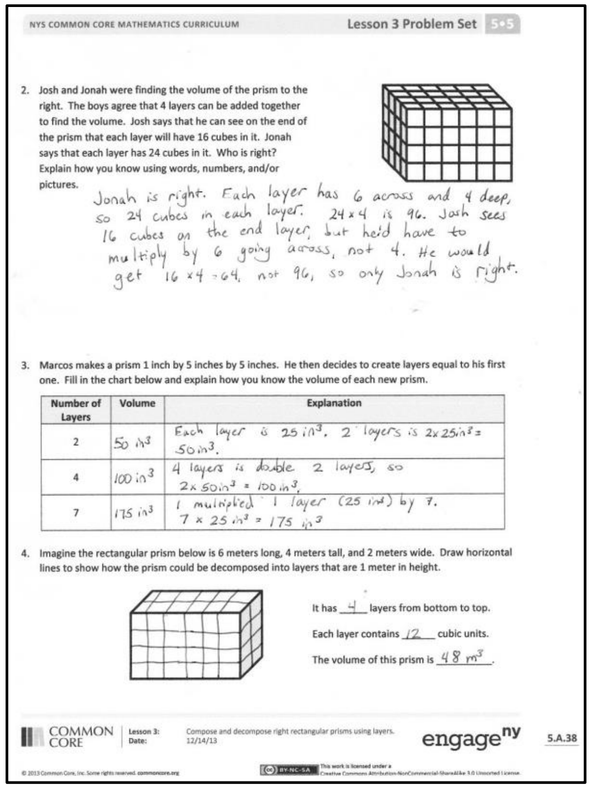
Lesson Objective: Compose and decompose right rectangular prisms using layers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1, how did you decide how to go about decomposing the prisms? Is there a different way or order in which you could have done it?
- Problem 4 uses meters instead of centimeters. What, if anything, did that change in how you drew your picture? How about in how you figured out the volume?
- What was Josh having a hard time visualizing in Problem 2? Which layers are easier for you to visualize? Which are the hardest? How can you make the hardest layers easier to see?
- At what point did you not need to model with the physical cubes anymore?
- How did the Application Problem connect to today's lesson? How are stacks of ice trays different from the prisms in the lesson?



NYS COMMON CORE MATHEMATICS CURRICULUM Lesson 3 Problem Set 5•5

2. Josh and Jonah were finding the volume of the prism to the right. The boys agree that 4 layers can be added together to find the volume. Josh says that he can see on the end of the prism that each layer will have 16 cubes in it. Jonah says that each layer has 24 cubes in it. Who is right? Explain how you know using words, numbers, and/or pictures.

Jonah is right. Each layer has 6 across and 4 deep, so 24 cubes in each layer. 24 x 4 is 96. Josh sees 16 cubes on the end layer, but he'd have to multiply by 6 going across, not 4. He would get 16 x 4 = 64, not 96, so only Jonah is right.

3. Marcos makes a prism 1 inch by 5 inches by 5 inches. He then decides to create layers equal to his first one. Fill in the chart below and explain how you know the volume of each new prism.

Number of Layers	Volume	Explanation
2	50 in^3	Each layer is 25 in^3. 2 layers is 2x25 in^3 = 50 in^3.
4	100 in^3	4 layers is double 2 layers, so 2x 50 in^3 = 100 in^3.
7	175 in^3	I multiplied 1 layer (25 in^3) by 7. 7 x 25 in^3 = 175 in^3

4. Imagine the rectangular prism below is 6 meters long, 4 meters tall, and 2 meters wide. Draw horizontal lines to show how the prism could be decomposed into layers that are 1 meter in height.

It has 4 layers from bottom to top.

Each layer contains 12 cubic units.

The volume of this prism is 48 m^3.

COMMON CORE Lesson 3: Compose and decompose right rectangular prisms using layers. Date: 12/14/13 engage^ny 5.A.38

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Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A # Correct _______

Solve.

1	$\frac{1}{5} \times 2 =$		23	$\frac{5}{6} \times 12 =$	
2	$\frac{1}{5} \times 3 =$		24	$\frac{1}{3} \times 15 =$	
3	$\frac{1}{5} \times 4 =$		25	$\frac{2}{3} \times 15 =$	
4	$4 \times \frac{1}{5} =$		26	$15 \times \frac{2}{3} =$	
5	$\frac{1}{8} \times 3 =$		27	$\frac{1}{5} \times 15 =$	
6	$\frac{1}{8} \times 5 =$		28	$\frac{2}{5} \times 15 =$	
7	$\frac{1}{8} \times 7 =$		29	$\frac{4}{5} \times 15 =$	
8	$7 \times \frac{1}{8} =$		30	$\frac{3}{5} \times 15 =$	
9	$3 \times \frac{1}{10} =$		31	$15 \times \frac{3}{5} =$	
10	$7 \times \frac{1}{10} =$		32	$18 \times \frac{1}{6} =$	
11	$\frac{1}{10} \times 7 =$		33	$18 \times \frac{5}{6} =$	
12	$4 \div 2 =$		34	$\frac{5}{6} \times 18 =$	
13	$4 \times \frac{1}{2} =$		35	$24 \times \frac{1}{4} =$	
14	$6 \div 3 =$		36	$\frac{3}{4} \times 24 =$	
15	$\frac{1}{3} \times 6 =$		37	$32 \times \frac{1}{8} =$	
16	$10 \div 5 =$		38	$32 \times \frac{3}{8} =$	
17	$10 \times \frac{1}{5} =$		39	$\frac{5}{8} \times 32 =$	
18	$\frac{1}{3} \times 9 =$		40	$32 \times \frac{7}{8} =$	
19	$\frac{2}{3} \times 9 =$		41	$\frac{5}{9} \times 54 =$	
20	$\frac{1}{4} \times 8 =$		42	$63 \times \frac{7}{9} =$	
21	$\frac{3}{4} \times 8 =$		43	$56 \times \frac{3}{7} =$	
22	$\frac{1}{6} \times 12 =$		44	$\frac{6}{7} \times 49 =$	

B Improvement ________ # Correct ________

Solve.

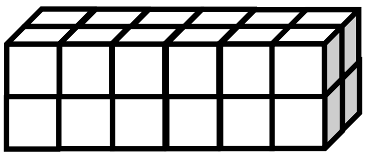
1	$\frac{1}{7}$ x 2 =		23	$\frac{3}{4}$ x 8 =	
2	$\frac{1}{7}$ x 3 =		24	$\frac{1}{5}$ x 15 =	
3	$\frac{1}{7}$ x 4 =		25	$\frac{2}{5}$ x 15 =	
4	4 x $\frac{1}{7}$ =		26	$\frac{4}{5}$ x 15 =	
5	$\frac{1}{10}$ x 3 =		27	$\frac{3}{5}$ x 15 =	
6	$\frac{1}{10}$ x 7 =		28	15 x $\frac{3}{5}$ =	
7	$\frac{1}{10}$ x 9 =		29	$\frac{1}{3}$ x 15 =	
8	9 x $\frac{1}{10}$ =		30	$\frac{2}{3}$ x 15 =	
9	3 x $\frac{1}{8}$ =		31	15 x $\frac{2}{3}$ =	
10	5 x $\frac{1}{8}$ =		32	24 x $\frac{1}{6}$ =	
11	$\frac{1}{8}$ x 5 =		33	24 x $\frac{5}{6}$ =	
12	10 ÷ 5 =		34	$\frac{5}{6}$ x 24 =	
13	10 x $\frac{1}{5}$ =		35	20 x $\frac{1}{4}$ =	
14	9 ÷ 3 =		36	$\frac{3}{4}$ x 20 =	
15	$\frac{1}{3}$ x 9 =		37	24 x $\frac{1}{8}$ =	
16	10 ÷ 2 =		38	24 x $\frac{3}{8}$ =	
17	10 x $\frac{1}{2}$ =		39	$\frac{5}{8}$ x 24 =	
18	$\frac{1}{3}$ x 6 =		40	24 x $\frac{7}{8}$ =	
19	$\frac{2}{3}$ x 6 =		41	$\frac{5}{9}$ x 63 =	
20	$\frac{1}{6}$ x 12 =		42	54 x $\frac{7}{9}$ =	
21	$\frac{5}{6}$ x 12 =		43	49 x $\frac{3}{7}$ =	
22	$\frac{1}{4}$ x 8 =		44	$\frac{6}{7}$ x 56 =	

Name ____ Date ____________________

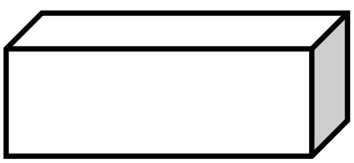
1. Use the prisms to find the volume.

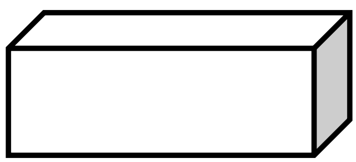
   - Build the rectangular prism pictured below to the left with your cubes, if necessary.
   - Decompose it into layers in three different ways, and show your thinking on the blank prisms.
   - Complete the missing information in the table.

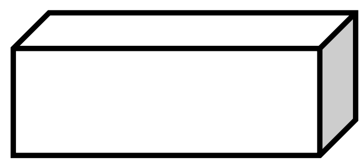
a.



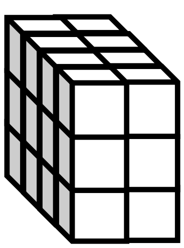
Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
		cubic cm
		cubic cm
		cubic cm



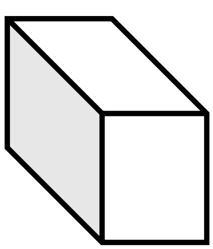


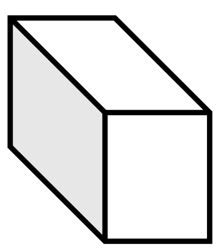


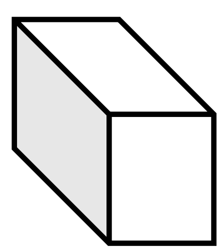
b.



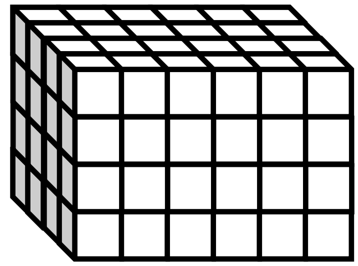
Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
		cubic cm
		cubic cm
		cubic cm







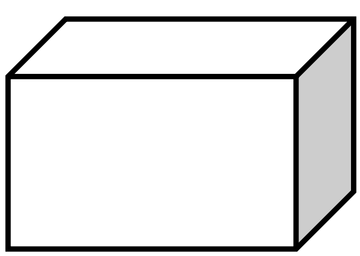
2. Josh and Jonah were finding the volume of the prism to the right. The boys agree that 4 layers can be added together to find the volume. Josh says that he can see on the end of the prism that each layer will have 16 cubes in it. Jonah says that each layer has 24 cubes in it. Who is right? Explain how you know using words, numbers, and/or pictures.



3. Marcos makes a prism 1 inch by 5 inches by 5 inches. He then decides to create layers equal to his first one. Fill in the chart below, and explain how you know the volume of each new prism.

Number of Layers	Volume	Explanation
2		
4		
7		

4. Imagine the rectangular prism below is 6 meters long, 4 meters tall, and 2 meters wide. Draw horizontal lines to show how the prism could be decomposed into layers that are 1 meter in height.



It has _____ layers from bottom to top.

Each layer contains ______ cubic units.

The volume of this prism is __________.

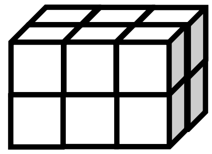
Name ____ Date ____________________

1. Use unit cubes to build the figure to the right and fill in the missing information.

   Number of layers: _______

   Number of cubes in each layer: ______

   Volume: ______ cubic centimeters

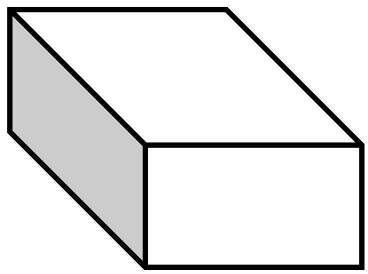


2. This prism measures 3 units by 4 units by 2 units. Draw the layers as indicated.

   Number of layers: 4

   Number of cubic units in each layer: 6

   Volume: ______ cubic centimeters



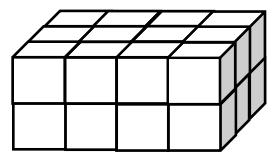
Name ____ Date ____________________

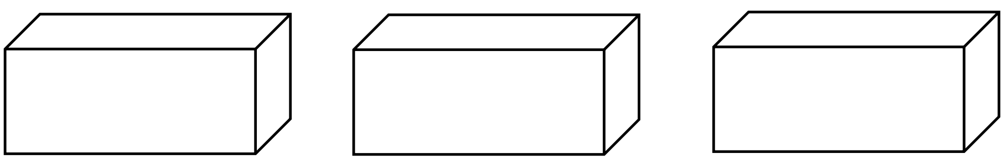
1. Use the prisms to find the volume.

- The rectangular prisms pictured below were constructed with 1 cm cubes.
- Decompose each prism into layers in three different ways, and show your thinking on the blank prisms.
- Complete each table.

a.

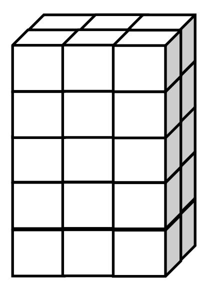
Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
		cubic cm
		cubic cm
		cubic cm

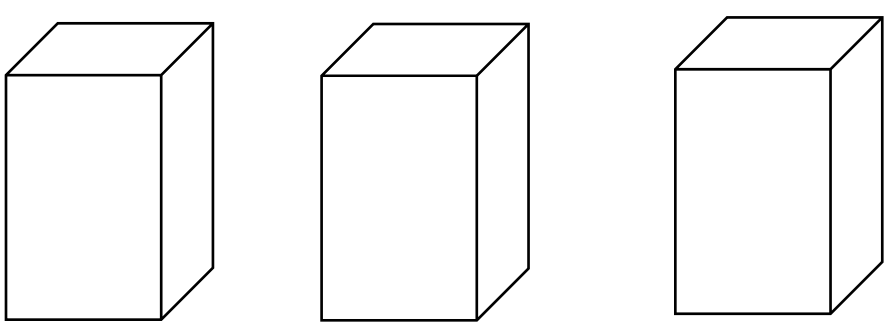




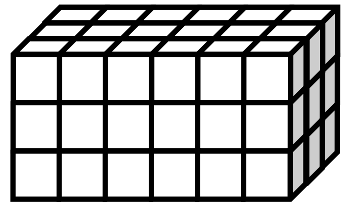
b.

Number of Layers	Number of Cubes in Each Layer	Volume of the Prism
		cubic cm
		cubic cm
		cubic cm





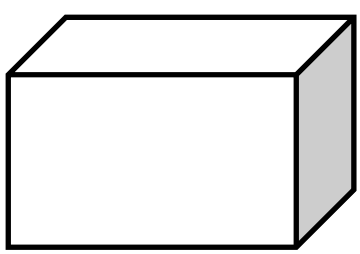
2. Stephen and Chelsea want to increase the volume of this prism by 72 cubic centimeters. Chelsea wants to add eight layers, and Stephen says they only need to add four layers. Their teacher tells them they are both correct. Explain how this is possible.



3. Juliana makes a prism 4 inches across and 4 inches wide but only 1 inch tall. She then decides to create layers equal to her first one. Fill in the chart below, and explain how you know the volume of each new prism.

Number of Layers	Volume	Explanation
3		
5		
7		

4. Imagine the rectangular prism below is 4 meters long, 3 meters tall, and 2 meters wide. Draw horizontal lines to show how the prism could be decomposed into layers that are 1 meter in height.



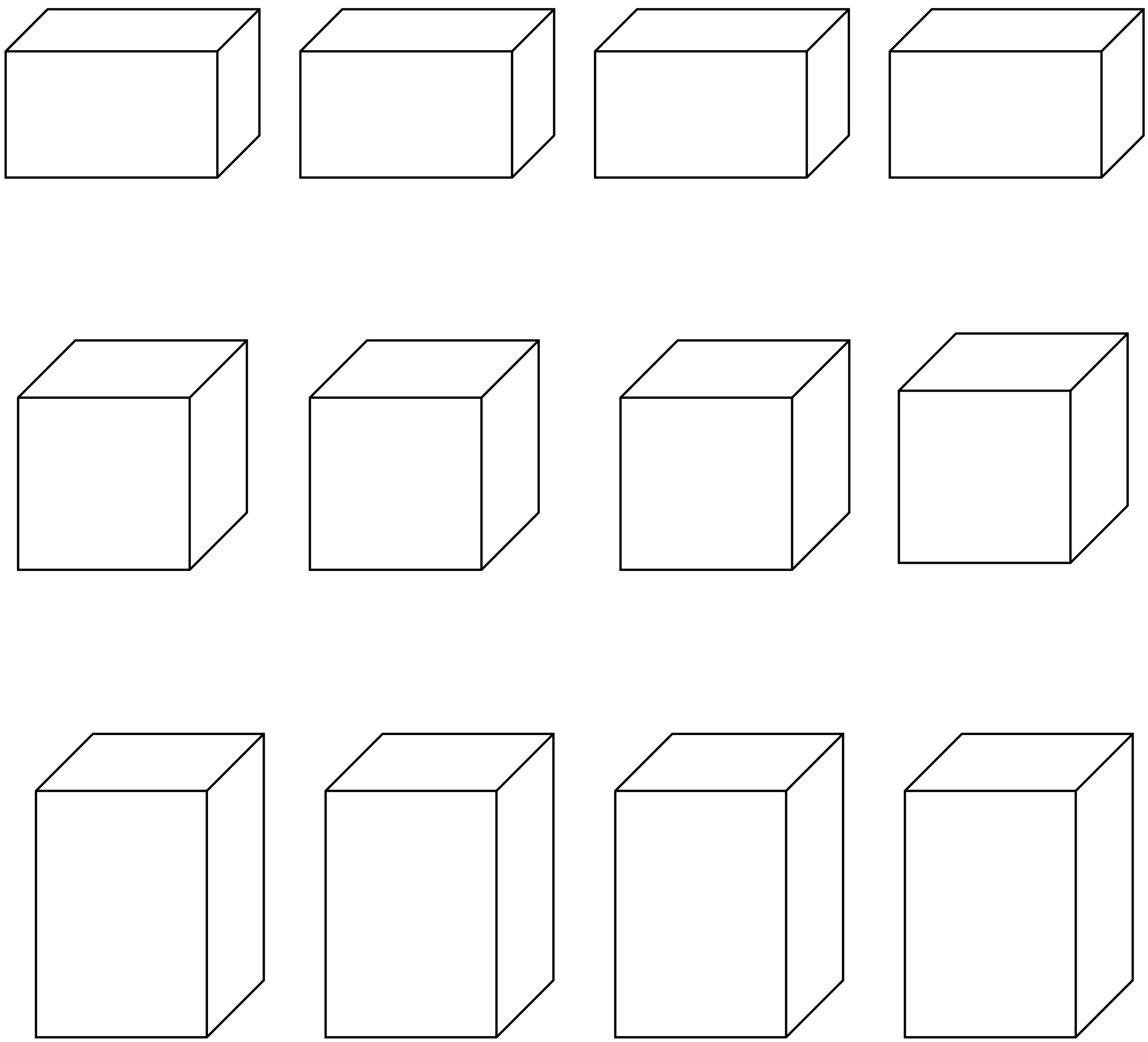
It has _____ layers from left to right.

Each layer contains ______ cubic units.

The volume of this prism is __________.

Name ____ Date ____________________

Use these rectangular prisms to record the layers that you count.



rectangular prism recording sheet